

Name of the course	:	B.Sc.(H)Mathematics
Unique Paper Code	:	32351101
Name of the Paper	:	C-1 Calculus (BMATH 101)
Semester	:	I
Duration	:	3hours
Maximum Marks	:	75

Attempt any **four** questions. All questions carry equal marks.

1. Sketch the graph of $f(x) = x^3 - 12x + 4$ by finding intervals of increase and decrease, critical points, relative extrema and concavity of the given function.

Find the n th derivative of

$$y = e^{2x}(\sin^3 x + \cos^2 x).$$

If $y = \log(x + \sqrt{x^2 + 1})$, prove that

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2y_n = 0.$$

7+5+6.75

2. Find

i. $\lim_{x \rightarrow \frac{\pi}{2}} (\tan x - \sec x)$

ii. $\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\log x} \right)$

iii. $\lim_{x \rightarrow 0} \left(\cot x - \frac{1}{x} \right)$

Sketch the parabola

$$\left(x - \frac{1}{2}\right)^2 = 2(y - 1)$$

and label the focus, vertex, axis and directrix.

Find the centre, vertices, foci and ends of minor axis of the ellipse

$$3x^2 + 4y^2 - 30x - 8y + 67 = 0.$$

6+6.75+6

3 (i) Find the volume of the solid that is generated by revolving the region bounded by the lines $y = x$, $y = 1$ and $x = 0$ about the x -axis.

(ii) Use cylindrical shells to find the volume of the solid generated when the region bounded by the curves $y = x$ and $y = x^2$ is revolved about the x -axis.

(iii) Find the arc length of the curve $f(x) = 3x^{\frac{3}{2}} - 1$ over the interval $[0,1]$. (7+6.75+5)

4 (i) Find the tangent vector and parametric equations for the tangent line to the graph of the vector function

$$\vec{F}(t) = e^{2t}\hat{i} + (t^2 - t)\hat{j} + (\log t)\hat{k}$$

at the point P corresponding to $t = 0.2$.

(ii) If $\vec{r}(t)$ is the position of a particle in plane at time t , find the time in the given interval when the velocity and acceleration are orthogonal, where

$$\vec{r}(t) = (t - \sin t)\hat{i} + (1 - \cos t)\hat{j}, 0 \leq t \leq 2\pi$$

(iii) A shell is fired from ground level with a muzzle speed of 280 ft/s and at an elevation of 45° from the ground level.

a) Find the maximum height attained by the shell.

b) Find the velocity and speed of the shell at impact.

(6+6+6.75)

5 Find horizontal asymptotes (if any) for the graph of

$$y = \left(\frac{x+1}{x+2}\right)^x$$

Trace the curve $r = 2\cos 3\theta$

A firm determines that x units of its products can be sold daily at p dollars per unit

where $x = 1000 - p$

The cost of producing x units per day is $C(x) = 3000 + 20x$

Find average revenue, profit function, maximum profit. What price per unit should be charged to obtain maximum profit. (6+6+6.75)

6 Find the curvature and radius of curvature for

$$r = 1 + \cos \theta \text{ at } \theta = \pi/2$$

Given \vec{v} and \vec{a} are velocity and acceleration (respectively) of a moving particle at a certain instant of time.

$$\vec{v} = 3\hat{i} - 4\hat{j}, \vec{a} = \hat{i} - \hat{j} + 2\hat{k}$$

Find tangential and normal components of velocity and acceleration, unit tangent vector and unit normal vector at this instant.

Evaluate

$$\int_0^{\pi} \sin^5 \theta \sqrt{\frac{1-\cos \theta}{1+\cos \theta}} d\theta$$

(6+7.75+5)

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